Contingent Sustainable Debt Restructuring with new Financing in the time of Covid 19: A Preliminary Theoretical Analysis

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**Abstract**

Should the consequences of the Covid 19 shock on low income countries be treated as a temporary shock or as one with potentially longer-term consequences and if so, what are the implications for debt restructuring? Should debt restructuring be contingent? Is new financing required to ensure sustainable debt restructuring? We provide a theoretical analysis of these issues. We begin by showing that a necessary and sufficient condition for a non-contingent debt restructuring proposal to be sustainable is that the interest rate at which the debt is restructured is lower than the expected rate of growth given the level of financing being proposed. We use this result to show that if the restructured debt is required to have positive present value, then there are conditions under which new financing will be required. We show that for any level of new financing, there is always a sustainable contingent debt restructuring proposal with positive expected present value. However, faced with a situation where non-contingent debt restructuring proposal is unsustainable, even with contingent debt restructuring, avoiding a debt write-down requires new financing.

1. **Introduction**

Even before the global negative shock resulting from the Covid 19 pandemic, in a recent report the World Bank (2020)) reports that total [emerging market developing economy debt reached an all-time high of almost 170 percent of GDP](http://pubdocs.worldbank.org/en/657041574888346651/Global-Economic-Prospects-January-2020-Topical-Issue-2-highlights.pdf) in 2018, an increase of 54 percentage points of GDP since 2010. Subsequently, the COVID-19 crisis led to a sudden collapse in capital flows to emerging and developing countries. The Institute of International Finance estimates that that there were portfolio outflows from emerging market countries amounted to nearly $100 billion over a period of 45 days starting in late February 2020 (IIF 2020). Moreover, the countries at high or moderate risk of debt distress are disproportionately fragile and conflict-affected states, commodity-dependent countries, and small States.

This state of affairs has lead to calls for an immediate payments standstill (with debt payments postponed in the short-term (see, for example, Bolton et.al. (2020)). As the World Bank Report referred to before points out, “Debt service suspension is a powerful, fast-acting measure that can bring real benefits to people in poor countries, particularly countries that don’t have the financial resources to respond to the coronavirus (COVID-19) crisis.”

The IMF and the World Bank have been proactive in implementing a debt standstill. The G20 has agreed to a debt service standstill on bilateral loans for a group of 76 low-income countries. Some private debt has also been rolled over (Eichengreen 2020). However, should the consequences of the Covid 19 shock on low income countries be treated as a temporary shock or as one with potentially longer-term consequences and if so, what are the implications for debt restructuring? Should debt restructuring be contingent (Eichengreen (2020))? Is new financing required to ensure sustainable debt restructuring?

In short note, we provide a theoretical analysis of this problem. We construct a simple model of debt restructuring in the face an unanticipated negative shock. Our focus is on sustainable debt restructuring. In our model, whether or not the unanticipated negative shock has permanent consequences depends on the additional financing (investment) proposed as part of the debt restructuring plan. To keep the exposition simple, we abstract away from issues of creditor coordination and debtor and creditor moral hazard. In later iterations of this paper, we will extend our analysis to incorporate these concerns.

We begin by examining the sustainability of a non-contingent debt restructuring proposal where the interest rate at which the debt is restructured is not contingent on recovery. We show that that a necessary and sufficient condition for a non-contingent debt restructuring proposal to be sustainable is that the interest rate at which the debt is restructured is lower than the expected rate of growth given the level of financing being proposed. We use this result to examine the sustainability of a debt standstill (with partial payments standstill) and a debt write down (with no new financing) and show that there are conditions under which new financing will be required to ensure a sustainable debt standstill and a sustainable debt write-down.

We then examine sustainable contingent debt restructuring. We show that for any level of financing, there is always a sustainable contingent debt restructuring proposal with positive expected present value. However, we also show that there cannot be a sustainable contingent debt restructuring proposal with the same expected present value as an unsustainable non-contingent debt restructuring proposal. Faced with a unsustainable non-contingent debt restructuring proposal, if the level of financing is taken as fixed, a sustainable contingent debt proposal must entail a debt write-down is expected terms. In such a scenario, avoiding a debt write-down will require new financing.

Section 2 is presents an analysis of sustainable non-contingent debt restructuring in the face of an unanticipated negative shock. Section 3 uses the analysis of Section 2 to examine the sustainability of a debt standstill and a debt write down. Section 4 examines sustainable contingent debt financing. The last section concludes.

1. **Sustainable Debt Restructuring in a simple model**

Take the case of a country embarking on a bond‐financed investment project, costing , which lasts only two periods. All the finance is supplied by external creditors who are promised returns of  in the first period and in the second period. So long as resources available cover these payments (i.e. cash flow in period 1 is greater than  and cash flow in period 2 is greater than , all is well and the project will run to completion.

Consider what happens if an unanticipated, exogenous shock (e.g. the Covid 19 pandemic ‘bad luck’) lowers the capacity to pay in period one below the amount that is due to bond holders under their contract i.e. where is the income in period 1 and is the minimum consumption I period 1.

Conditional on the negative shock, there is uncertainty about whether project net worth will be unchanged. This will depend on beliefs about the prospect of recovery i.e. whether the negative shock will turn out to be temporary or permanent.

Assuming that is advanced to the sovereign for reinvestment in where is the additional investment that determines the probability of recovery (with probability the shock has permanent negative consequences) where recovery implies that at , the total capacity to pay is (without recovery, available resources are with ).

Throughout the paper, we will assume that is a non-decreasing function of investment whenever with .

We now define, formally, a debt restructuring proposal:

**Definition 1.** The pair  is a debt restructuring proposal.

Consider what happens if creditors (as a group) choose to advance to the sovereign via a debt restructuring proposal . Given , the present value of the refinanced debt will be . In expected terms, the total resources available is . Hence, for restructuring to be sustainable (in expected terms), we must have

…(1)

Given a debt restructuring proposal , note that is the total amount of money loaned to the debtor for investment. The expected return is . Hence, the ratio of the expected return to amount invested is

and the expected rate of growth is .

The following proposition provides a basic characterization for a sustainable debt restructuring proposal ():

**Proposition 1**: *A debt restructuring proposal () is sustainable if and only if i.e. the interest rate at which debt is refinanced is less than the expected rate of growth.*

**Proof.** Note that the LHS of (1) is increasing in . Let . Hence, we compute the roots of the equation

……..(2)

and check these roots are all less than .

Note that the roots of the equation (2) are the same as the roots of the equation

By computation, note that this equation has two real roots one positive and one negative. Ignoring the negative root (as the interest rate has to be non-negative), the expression for the positive root is:

We want to show that . The argument proceeds by contradiction. So suppose to the contrary that

It follows that

so that

Hence,

and

Therefore, by cancelling the common term on both sides of the inequality, we must have that

a contradiction as, by construction, . Hence we must have .

The preceding proposition leads us to the following definition:

**Definition 2:** A debt restructuring proposal is sustainable whenever .

With the definition of a sustainable debt restructuring proposal in place, in next section, we evaluate a number of different debt restructuring proposals that have been proposed to deal with negative shock of Covid 19.

1. **Evaluating Debt Standstill and Debt Write-Down**

A key issue, often implicit, in current discussions around sovereign debt restructuring in the face of Covid 19 is whether a debt restructuring proposal should treat the shock as temporary or as one with potentially permanent consequences.

In our model, with our focus on debt sustainability, the permanent consequences of the shock is endogenous i.e. it depends on the actual debt restructuring plan being proposed or implemented.

We use our results to evaluate the sustainability of two debt restructuring proposals: a debt standstill and a debt write down. In each case, we characterize the conditions under which debt sustainability is restored and specifically, where restoring debt sustainability requires new investment.

However, before we turn to these two proposals, we need a preliminary result.

The next proposition provides a characterization of sustainable debt restructuring proposal of the when the growth function is elastic:

**Proposition 2**: *Suppose for all . Then, given (respectively, ), there exists an upper bound (resp. ) such that is a sustainable debt restructuring proposal if and only if with (resp. with ).*

**Proof**. By Proposition 1, a debt restructuring proposal *()* is sustainable if and only if . Consider the equation

By lemma 1, For a given value of , when , there is a unique solution to the preceding equation which is increasing in . For a given value of , when , there is a unique solution to the preceding equation which is increasing in . The conclusion follows.

**Debt Standstill**

Consider the proposal that there should be a debt standstill for Covid 19. This proposal would temporarily freeze all or some repayments on existing debt with the amount owed rolled over to the following period at the original interest rate and all payments temporarily suspended.

In terms of the model presented here this means that and  (if all payments are suspended); if some (but not all payments) are being suspended then .

**Definition 3.** A debt standstill with some payments suspended is a debt restructuring proposal with and .

For such a debt standstill to be sustainable (in expected terms) the following inequality must hold for some :

…(3)

By Proposition 1, note that a debt standstill is sustainable if and only if form some . Moreover, by Proposition 2, when , a debt standstill is sustainable if and only if  (equivalently, ).

Therefore, if (equivalently, *)*, restoring sustainability requires either or or both.

The implication is that there are conditions under which, for a debt restructuring proposal to be sustainable, it has to go beyond a debt standstill requiring either that and or both.

**Debt Write-Down**

Note that the present value of the original debt is . So a debt write-down would entail haircut on the present value of the original debt.

In terms of the model presented here this means that and i.e. there is no additional lending but a permanent write down in the present value of the debt.

**Definition 4.** A debt write-down is a debt restructuring proposal with (otherwise, the restructured debt will have negative present value) and .

As , for a debt write-down to be sustainable (in expected terms) the following inequality must hold:

…(4)

As,by computation note that .

Hence, by Proposition 1, *a debt write-down is sustainable if and only if .*

Moreover, by Proposition 2, if, then a debt write-down is sustainable if and only if . If (equivalently,*)*, then sustainability requires .

The implication is that when future growth prospects are substantially diminished following a negative shock such as Covid 19, then the debt write down must be large enough so that the present value of restructured is debt is close to zero *and* additional investment is made in the form of a grant or at an interest rate close to zero.

1. **Contingent Debt Restructuring**

In our analysis so far, the interest rate at which debt is not contingent on future state of the world. This has the consequence that the sustainability constraint holds as in expected terms so that there could be a future state of the world where the restructured debt may not be repaid.

To ensure that sustainability holds in all future states if the world, we need to consider contingent debt restructuring where the interest rate paid on sovereign debt is contingent on the state of the world that prevails. For example, the interest rate could be linked to future GDP or future export earnings.

Formally, we define a contingent debt restructuring proposal as follows:

**Definition 5.** The triple  is a contingent debt restructuring proposal.

For clarity of exposition, we will refer a debt structuring proposal as an non-contingent debt restructuring proposal.

Consider what happens if creditors (as a group) choose to advance to the sovereign via a contingent debt restructuring proposal. Debt sustainability requires that the following two inequalities simultaneously hold:

….(5a)

…(5b)

The contingent return to the amount invested is

and the contingent rate of growth is  and .

The following proposition provides a basic characterization for a sustainable contingent debt restructuring proposal :

**Proposition 3**: *(a) Consider a* *sustainable* *contingent* *debt restructuring proposal* *. Then, there is* *an* *non-contingent debt restructuring proposal with* *that is also sustainable and has positive expected present value.* *(b) Moreover, suppose the non-contingent debt restructuring proposal is not sustainable.* *Then, there is no sustainable contingent debt restructuring proposal . with .*

**Proof.** The proof of part (a) is as follows. We show that that a contingent debt restructuring proposal is sustainbale if and only if and only if both *and .* We provide a proof for ; the proof for  is omitted as it is analogous. Note that the LHS of (5a) is increasing in . Let . Hence, we compute the roots of the equation

……..(6)

and check these roots are all less than .

Note that the roots of the equation (6) are the same as the roots of the equation

By computation, note that this equation has two real roots one positive and one negative. Ignoring, as before, the strictly negative root, the expression for the other root is:

Evidently, so that there is strictly positive root. We want to show that the strictly positive root . The argument proceeds by contradiction. So suppose to the contrary that

It follows that

so that

Hence,

and

Therefore, by cancelling the common term on both sides of the inequality, we must have that

a contradiction as, by construction, . Hence, we must have that  and . Next, we show that for each , a sustainable contingent debt proposal with positive expected present value exists. Note that, by construction, both:

and

Therefore, for some , there exists pair such that both

and

Now:

Therefore, by Proposition 1, the non-contingent debt restructuring proposal with is sustainable and by construction has positive expected present value. The proof of Part (b) follows by contradiction. Suppose, to the contrary, *there is a sustainable contingent debt restructuring proposal . with .* For any sustainable contingent debt contract we must have  and  so that , a contradiction. The conclusion follows.

Although part (a) of Proposition 3 shows that there is always a contingent debt restructuring proposal with positive expected present value, part (b) shows that there cannot be a sustainable contingent debt restructuring proposal with the same expected present value as an unsustainable non-contingent debt restructuring proposal.

Hence, faced with a unsustainable non-contingent debt restructuring proposal, if the investment is taken as fixed, a sustainable contingent debt proposal must entail a debt write-down is expected terms. So avoiding a debt write-down requires new financing.

1. **Conclusion**

We provide a theoretical analysis of this problem. We begin by examining the sustainability of a non-contingent debt restructuring proposal where the interest rate at which the debt is restructured is not contingent on recovery. We show that that a necessary and sufficient condition for a non-contingent debt restructuring proposal to be sustainable is that the interest rate at which the debt is restructured is lower than the expected rate of growth given the level of financing being proposed. We use this result to show that if the restructured debt is required to have positive present value, then new financing could be required with a debt standstill and a debt write-down. Moreover, we show that for any level of financing, there is always a sustainable contingent debt restructuring proposal with positive expected present value. However, we also show that there cannot be a sustainable contingent debt restructuring proposal with the same expected present value as an unsustainable non-contingent debt restructuring proposal. Faced with a unsustainable non-contingent debt restructuring proposal, if the level of financing is taken as fixed, a sustainable contingent debt proposal must entail a debt write-down is expected terms: avoiding a debt write-down requires new financing.

In later iterations of this paper, we will extend our model to examine issues of creditor coordination and debtor and creditor moral hazard as well as provide an empirical analysis to back our theoretical analysis.

**References**

Bolton, P, L Buchheit, P-O Gourinchas, M Gulati, C-T Hsieh, U Panizza and B Weder di Mauro (2020), “[Born Out of Necessity: A Debt Standstill for COVID-19](https://cepr.org/sites/default/files/policy_insights/PolicyInsight103.pdf)”, CEPR Policy Insight no 103.

Eichengreen, B. (2020), “Managing the Coming Global Debt Crisis”, <https://www.project-syndicate.org/commentary/managiing-coming-global-debt-crisis-by-barry-eichengreen-2020-05?barrier=accesspaylog>

IIF – Institute of International Finance (2020), “[The COVID-19 Cliff](https://www.iif.com/Portals/0/Files/content/1_IIF_Capital%20Flows%20Tracker_April.pdf)”, IIF Capital Flows Tracker, April.

World Bank, (2020), “Debt Service Suspension and COVID-19”, see <https://www.worldbank.org/en/news/factsheet/2020/05/11/debt-relief-and-covid-19-coronavirus>